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Math 131 - Spring 2024 - Common Final Exam, version A

Print name:_____

Print instructor's name:_____

Directions:

- This exam has 14 questions worth a total of 100 points.
- Fill in your name and instructor's name above.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- You may use a calculator which cannot connect to the internet. The use of any notes or electronic devices other than a calculator is prohibited.

Good luck!

Question:	1	2	3	4	5	6	7	8
Points:	6	5	13	11	5	6	6	8
Score:								

Question:	9	10	11	12	13	14	Total
Points:	10	8	5	4	5	8	100
Score:							

1. After putting money into an investment fund for *t* months, your balance is *B* dollars, where $B = g(t)$. Give a one-sentence interpretation of each statement below. You must include units in your interpretation.

   a. (2 points) $g(20) = 1345$

   b. (2 points) $g'(20) = -12$

   c. (2 points) $\int_0^{20} g'(t)dt = 155$

2. (5 points) The revenue (in thousands of dollars) generated by a company's service is found using the function $R(q) = \sqrt{e^q + 3q}$ where *q* is the number of times the service is provided to a client. Using calculus, estimate the additional revenue generated by increasing *q* one unit at $q = 5$. Provide units and round your answer to two decimal places.

3. Let $f(x) = x^3 - 3x^2 + 1$ have domain $(-\infty, \infty)$.

   a. (2 points) Find $f'(x)$.

   b. (2 points) Find the x-coordinates of all critical points of $f(x)$.

   c. (3 points) Using either the first or second-derivative test, find and classify all critical points of $f(x)$ as a local maximum, local minimum, or neither. You must show your work.

   d. (3 points) Does $f(x)$ have a global maximum? Does $f(x)$ have a global minimum? Explain your answers using a complete sentence for each.

   e. (3 points) Use calculus to find the x-coordinates of any inflection points. Justify your answer in a complete sentence

4. The table below gives several values of a continuous, invertible function $f(x)$. Assume that the domain of both $f(x)$ and $f'(x)$ is the interval $(-\infty, \infty)$.

x	0	4	8	12	16	20	24	28	32
$f(x)$	-9	-5	-1	4	5	8.5	11	13.5	16

a. (3 points) Evaluate each of the following.
   i. $f(f(32))$

   ii. $f^{-1}(11)$

   iii. $f^{-1}(f(4)+4)$

b. (2 points) Compute the *average rate of change* of f on the interval $12 \le x \le 24$. Show your work.

c. (2 points) Estimate $f'(20)$. You must show your work.

d. (2 points) Suppose $f'(24) = 0.6$. $Estimate\ f(23)$. Show your work.

e. (2 points) If $f''(24) = -0.3$, do you expect your estimate in (d) to be an *overestimate* or an *underestimate*? Explain.

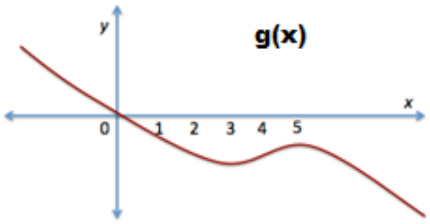
5. (5 points) Evaluate the following definite integral exactly using the Fundamental Theorem of Calculus. Show your work. A calculator solution will earn no credit.

$$\int_0^2 (4x + 2e^x)dx$$

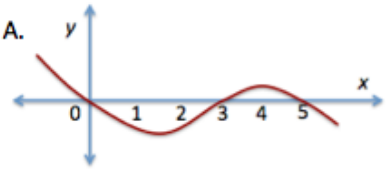
6. (6 points) Let $\frac{dy}{dt} = 4 + 3\sin(t)$. Find a formula for *y* in terms of *t* with the initial condition *y* = 7 when *t* = 0. Show your work.

7.

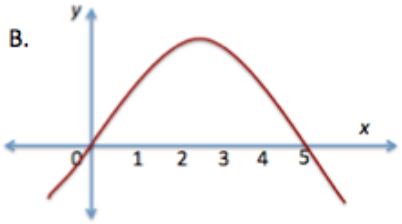
a. (3 points) The graph of $y = g(x)$ is shown here. Which one of the graphs below it could be the graph of its derivative, $g'(x)$? **Choose one and write the letter in the box below the graphs.**



A.



B.



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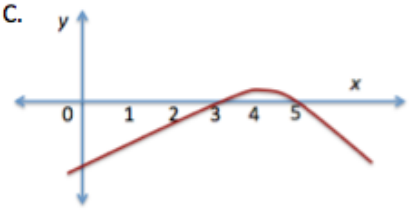
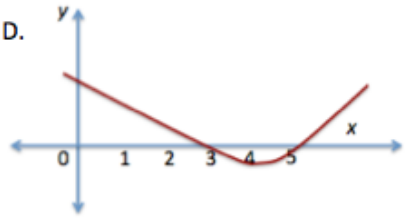
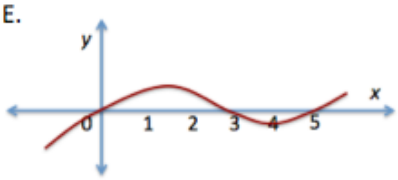


D.



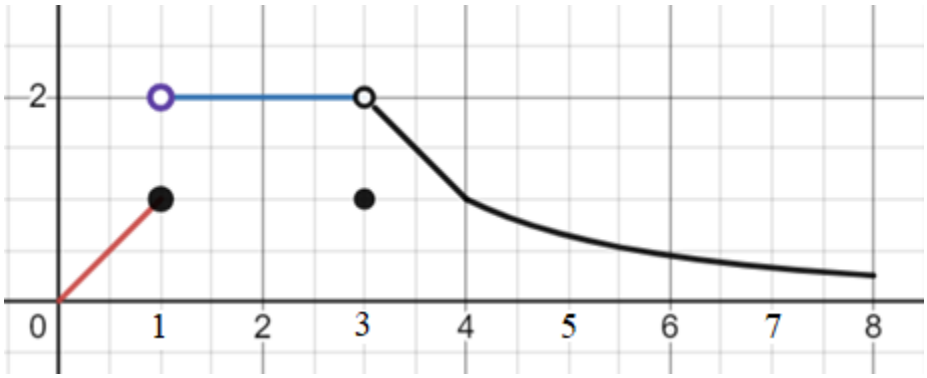
E.



The graph of $g'(x)$ is ☐

b. (3 points) Provide an explanation for your choice in part (a). State clearly why your choice is the only possible answer.

8. Consider the graph of $y = f(x)$ given below with domain $[0, 8]$.



a. (2 point) At which of the following points does $\lim_{x \to p} f(x)$ exist?

Circle all correct answers or write NONE.

p = 1 p = 2 p = 3 p = 4 p = 5 p = 6 p = 7

b. (2 point) What is the value of $\lim_{x \to 1^-} f(x)$?

Circle all correct answers.

0 1 2 3 DNE

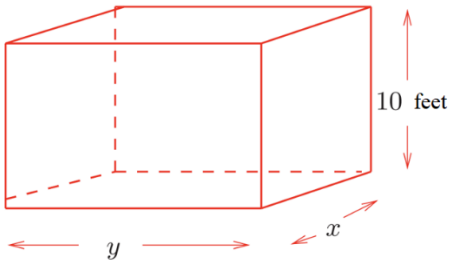
c. (2 point) At which of the following points is f continuous?

Circle all correct answers or write NONE.

p = 1 p = 2 p = 3 p = 4 p = 5 p = 6 p = 7

d. (2 points) Find $f'(3.5)$. Show your work.

9. Pandora has decided to build a display tank to house her carnivorous plants. She wants five sides to be made of glass and the top to be open. She also demands that the height be 10 feet.



Let the width be *x* (in feet), length be *y* (in feet), and the volume be *V*.

a. (2points) Express the volume of the box in terms of *x* and *y*.

b. (2 points) Write a formula for the total surface area, *A*, of the glass needed to build this box as a function of *x* and *y*.

c. (3 points) Pandora decides she wants the total volume to be 1000 cubic feet. Express the surface area, *A*, as a function of a single variable.

d. (3 points) Given that the glass needed is very expensive, using calculus, find *x* and *y* that minimize the surface area needed to make the box. Show all work.

10. Suppose that we wish to calculate the area under the curve $h(x) = \ln x$ over the interval [2, 6].

    a. (3 points) Approximate this area using a left-endpoint sum (i.e. Riemann left sum) with $n = 4$. Express your answer to the nearest tenth. Show your work. *(*An answer produced using only a calculator will earn no credit.)

    b. (2 points) Is your answer from part (a) an overestimate or an underestimate of the true value of the area? Explain your answer in a complete sentence.

    c. (3 points) Given that $H(x) = x \ln x - x$ is an antiderivative of $h(x) = \ln x$, use the Fundamental Theorem of Calculus to find the exact area beneath the curve $h(x) = \ln x$ over the interval [2, 6].

11. (5 points) Given the function *q(x)* below, use calculus to find a *q'(0)*. Simplify your answer fully. You must show all work for full credit.

$$q(x) = \frac{2 \sin(x)}{x^2 + 3}$$

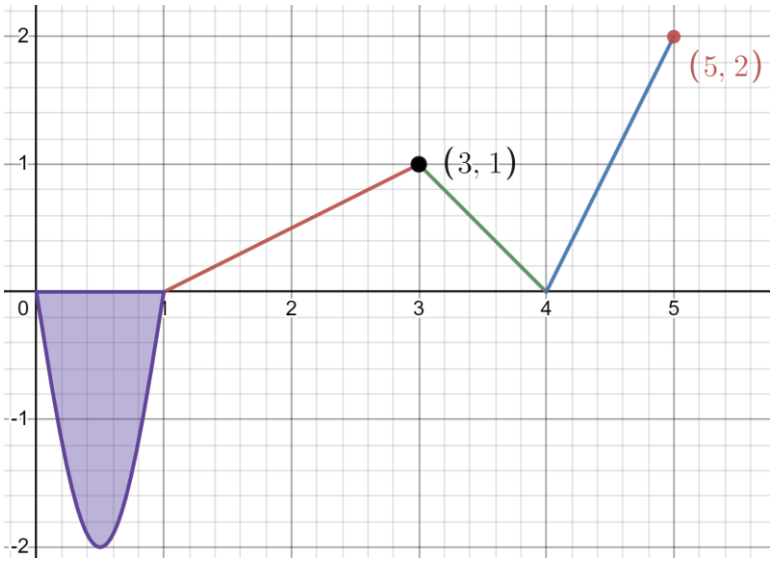
12. (4 points) Let $A(x) = x^x$

Use the limit definition of the derivative to write an explicit expression for $A'(4)$. Your answer should not involve the letter *A*. Do not attempt to evaluate or simplify the limit. Please write your final answer in the answer box provided below.

$A'(4) =$

13. (5 points) The number of people being vaccinated in a small city is steadily increasing. Let $P(t) = 5t \arctan(t)$ represent the total number vaccinated (in thousands of people) over *t* weeks. What is the rate of change of vaccinated people at the 8-week mark? You must use calculus and show all work. Include units and round your final answer to 2 decimal places.

14. Below is the graph of a function $y = f(x)$.



Assume that the area of the shaded region is 1.6.

a. (4 points) Find $\int_0^5 f(x)dx$. Show your work.

b. (4 points) Find $\int_1^3 \big(3 - 2f(x)\big)dx$. Show your work.

Elementary Tools from Algebra and Geometry

Quadratic Formula: $ax^2 + bx + c = 0 \quad \Longrightarrow \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Rectangle Area = base × height

Surface area of a 3D shape = sum of all side areas

Volume of a rectangular box = length × width × height.

Five derivative rules for operations on functions.

Constant Multiples: $\dfrac{d}{dx}\Big(cf(x)\Big) = cf'(x)$

Sums & Differences: $\dfrac{d}{dx}\Big(f(x) \pm g(x)\Big) = f'(x) \pm g'(x)$

Product Rule: $\dfrac{d}{dx}\Big(f(x) \cdot g(x)\Big) = f'(x)g(x) + f(x)g'(x)$

Quotient Rule: $\dfrac{d}{dx}\Big(\dfrac{f(x)}{g(x)}\Big) = \dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Chain Rule: $\dfrac{d}{dx}\Big(f(g(x))\Big) = f'(g(x)) \cdot g'(x)$

Ten derivative rules for functions

Derivative of a Constant: $\dfrac{d}{dx}(c) = 0$

The Power Rule: $\dfrac{d}{dx}\Big(x^n\Big) = nx^{n-1}$

Exponential Functions: General Case: $\dfrac{d}{dx}\Big(a^x\Big) = a^x \cdot \ln(a)$ Special Case: $\dfrac{d}{dx}\Big(e^x\Big) = e^x$

Three Trigonometric Rules:

$\dfrac{d}{dx}\Big(\sin(x)\Big) = \cos(x)$

$\dfrac{d}{dx}\Big(\cos(x)\Big) = -\sin(x)$

$\dfrac{d}{dx}\Big(\tan(x)\Big) = \sec^2(x) = \dfrac{1}{\cos^2(x)}$

Three Inverse Function Rules:

$\dfrac{d}{dx}\Big(\ln(x)\Big) = \dfrac{1}{x}$

$\dfrac{d}{dx}\Big(\arctan(x)\Big) = \dfrac{1}{1+x^2}$

$\dfrac{d}{dx}\Big(\arcsin(x)\Big) = \dfrac{1}{\sqrt{1-x^2}}$

General Antiderivative Rules

If k is a constant $\displaystyle\int k\,dx = kx + C$

$\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, when $n \neq -1$

$\displaystyle\int a^x\,dx = \frac{a^x}{\ln(a)} + C$

$\displaystyle\int e^x\,dx = e^x + C$

$\displaystyle\int \cos(x)\,dx = \sin(x) + C$

$\displaystyle\int \sin(x)\,dx = -\cos(x) + C$

$\displaystyle\int \sec^2(x)\,dx = \tan(x) + C$

$\displaystyle\int \frac{1}{x}\,dx = \ln(|x|) + C$

$\displaystyle\int \frac{1}{1+x^2}\,dx = \arctan(x) + C$

$\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C$